Unit 4

L1 Modes of Representing a Relation

There are different ways of representing a relation:

1. Verbal/Written:

Ex: A repairman charges $30 per hour plus $60 for his travel expenses.

2. Rule/Equation:

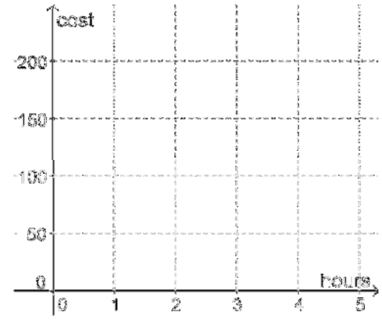
Ex: x: hours (indep. Var.) y: cost in $ (dep. Var.)

$y = 30x + 60$ or $C(h) = 30h + 60$

3. Table of Values:

Hours	0	1	2	3	4	5
Cost						

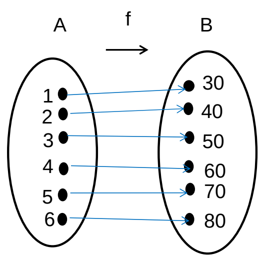
4. Cartesian graph :



Remarks:

1. The indep. Var. x goes on the Horizontal axis; the dep. Var. y goes on the vertical axis.
2. Choose an appropriate scale for the x and y-axis.
3. We can break the axis if the graph starts up too high.
4. Remember to label the axis, the scales, and put a title.

Example 1: A ferry ensures the transportation from a town to an island. The rates are the following: $20 per car and $10 per occupant in the car. No car is accepted without an occupant and there is a maximum of 6 occupants per car.



Example 2: Anna works as a dental hygienist in a clinic. Her hourly wage is $22. In this situation consider the following two variables: the number of hours worked in a week and her salary.

Example 3: A water reservoir contains 1000 liters of water. A pump is activated to empty the reservoir at a rate of 50 liters per minute. Consider the relation between the variable “elapsed time” and the variable “quantity of water left in the reservoir”.

Practice:

4.1 # 1,2

Unit 4

L2 Functions Vs. Relations

Definition:

A *relation* is a rule which takes an input and may/or may not produce **multiple** outputs.

A *function* is a special relation where every input has **at most** one output.

Notation:

Relation: $y = 2x + 1$

Read it "f of x": the brackets here DO NOT MEAN MULTIPLY!!

Function: $f(x) = 2x + 1$

Ex 1: find $f(7) =$

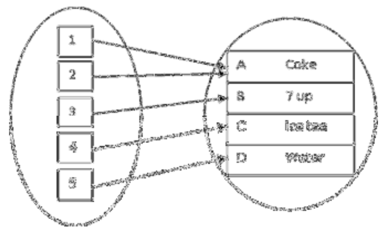
$=$

find $f(-5) =$

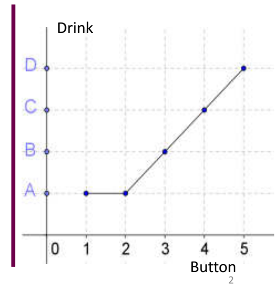
$=$

Consider 2 machines. **Machine A:**

Buttons	Drink
Push **IN** to order	Comes **OUT**



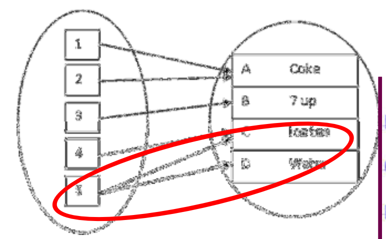
Vertical line test (VLT)



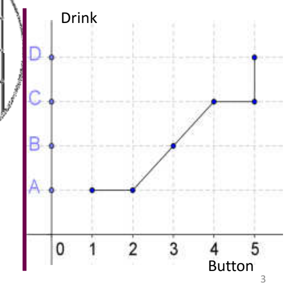
A FUNCTIONING MACHINE

Machine B:

Buttons	Drink
Push **IN** to order	Comes **OUT**



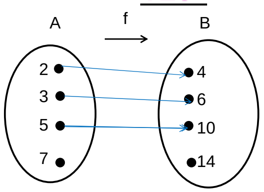
Vertical line test (VLT)



A Not FUNCTIONING MACHINE

(1) Mapping Diagram

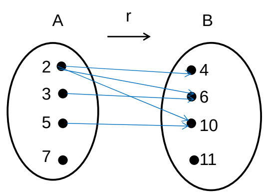
Function ____



X-values
Independent
Input
Source
Antecedent
Domain

Y-values
Dependent
Output
Target
Image
Range

Function ____



(2) Table of Values

Function ____

x	y
0	3
1	4
3	6
5	8

Each x value has only ONE y value.

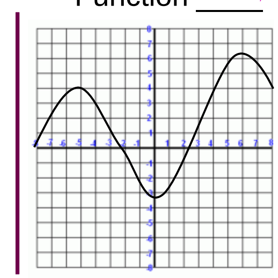
Function ____

x	y
1	0
2	1
4	3
4	5

An x value has MORE THAN ONE y value

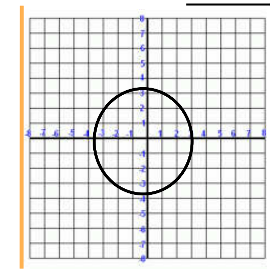
(3) Graph - Vertical Line Test (VLT)

Function ____



A vertical line touches the curve at ONLY 1 spot at a time

Function ____



A vertical line touches MORE THAN 1 spot at a time.

Ex 2: Evaluate the following functions

a) $f(x) = 3x + 5$ find $f(3)$

b) $g(x) = x^2 + 1$ find $g(6)$

c) $h(x) = (x-2)(x-5)$ find $h(6)$

Example 3

Consider the function f represented on the right. Determine if the following statements are true or false.

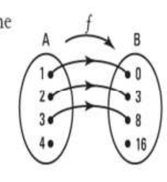
a) $f(0) = 1$ ____________

b) 3 is the image of 2 ______

c) 3 is the antecedent of 8 ____

d) $f(4)$ does not exist______

e) $f(x) = x^2 - 1$ __________



Example 4

Find the rule of the functions with the following graphs.

a) $f = \{(0, 0), (1, 1), (2, 4), (3, 9)\}$. ______

b) $g = \{(0, 0), (1, 2), (2, 4), (3, 6)\}$. ______

c) $h = \{(0, 1), (1, 3), (2, 5), (3, 7)\}$. ______

d) $u = \{(0, 1), (1, 2), (2, 5), (3, 10)\}$. ______

Practice:
4.2 # 1

Unit 4

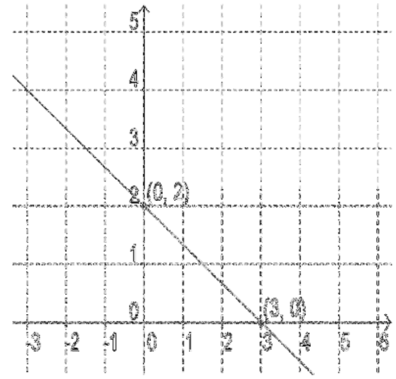
L3 Intercepts

- The point where a line intersects with an axis is called an **intercept**.
- Note that a horizontal line has no x-intercept,
  and a vertical line has no y-intercept
- A function can have **many x intercepts,** aka zeros,
  but **only one y intercept**, aka initial value.

Ex 1: What are the x-intercept and the y-intercept of this line

- x-intercept is 3
- note that the y coordinate there is 0

- y-intercept is 2
- note that the x coordinate there is 0



Ex 2: Calculate the x-intercept and the y-intercept for the line $y = 2x - 3$

- x-intercept: let y = 0 in the equation

  $0 = 2x - 3$

  $3 = 2x$

  $x = 1.5$

- y-intercept: let x = 0 in the equation

  $y = 2(0) - 3$

  $y = -3$

or we could read the **constant** from the rule of the line

Ex 3: Calculate the x-intercept and the y-intercept for the line $y = 2x + 7$

- x-intercept: let y = 0 in the equation

  $2x + 7 = 0$

  $2x = -7$

  $x = -3.5$

- y-intercept: let x = 0 in the equation

  $y = 2(0) + 7$

  $y = 7$

Ex 4: Complete the table of values for the line $3x + 4.5y + 9 = 0$

X	Y
0	-2
-3	0

- If x = 0 then $4.5y + 9 = 0$

  $4.5y = -9$

  $y = -2$

- if y = 0 then $3x + 9 = 0$

  $3x = -9$

  $x = -3$

Practice:
4.3 # 1

Unit 4

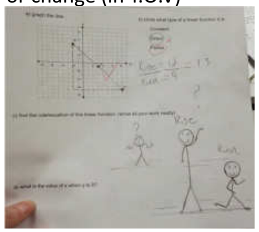
L4 Rate of change

- Rate of change (R.O.C) is also known as Rate of variation, Slope, and Rise over Run
- Given points $A(x_1,y_1)$ & $B(x_2,y_2)$ on a line, then the rate of change between points A & B is :

R.O.C. = $\frac{\Delta y}{\Delta x}$; Δ: difference or change (in T.O.V)

= $\frac{y_2 - y_1}{x_2 - x_1}$ given 2 points

or = $\frac{rise}{run}$ given a graph

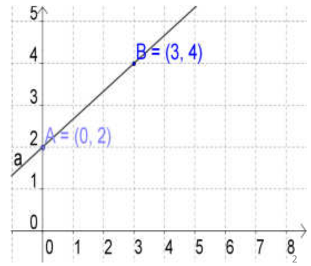


Ex 1: Given points A(0,2) and B(3,4)

- R.O.C. = $\frac{\Delta y}{\Delta x}$

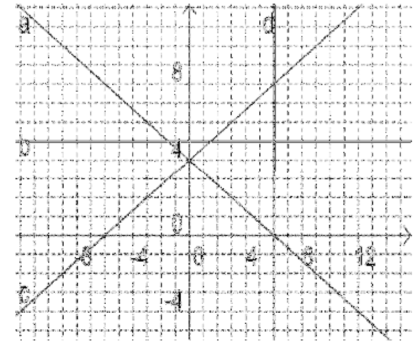
X	Y
0	2
3	4

- Or Count $\frac{rise}{run}$ =



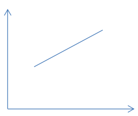
Ex 2: Find the slope of each line on the grid:

- Slope of line:
- a:
- b:
- c:
- d:

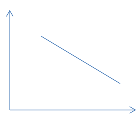


- On a straight line the ROC is constant (the same) no matter which 2 points we pick.
  - Where as on a curve the ROC changes (is not constant) along the curve.
- 4 possible slopes are noted:

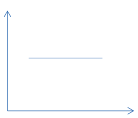
1. Positive slope :



2. Negative slope:



3. Zero slope:



4. Undefined slope:

Ex 3: Find the slope of the following:

a) A(1,3) B(2,5)

$\frac{rise}{run} = \frac{y_2-y_1}{x_2-x_1}$

=

=

=

b) C(-1,5) D(4,8)

$\frac{rise}{run} = \frac{y_2-y_1}{x_2-x_1}$

Find the value of c

Ex 4: The slope is 2
G(0,-5) and B(c,3)

Ex 5: The slope is 1/2
S(-1,2) and T(c,6)

Find the rate of Change for the Following

Ex 6: Jen earns \$220 for 20 hours of work. For 10 hours of work she earns \$120. What is her hourly rate?

Practice:

4.4 # 1-3

WS #1 Finding the Rate of change

Unit 4

L5 Finding the Rule of the Linear function

A straight line always follows the RULE

$$y = a\,x + b$$

Where:

- y → is the Dependent variable
- x → is the Independent variable
- a → is the R.O.C.(slope)
- b → is the initial value (y-intercept)

Steps to Finding the RULE given 2 points

Step 1: Find the slope using $\boxed{a = \frac{y_2 - y_1}{x_2 - x_1}}$

Step 2: Find the y-intercept (b) by plugging the (x_1, y_1) coordinates into $\boxed{b = y_1 - ax_1}$

Step 3: State the final equation.
$y = ax + b$

Ex1: Find the rule of the line going through (-6,5) & (-4, 6)

Step 1: Find a

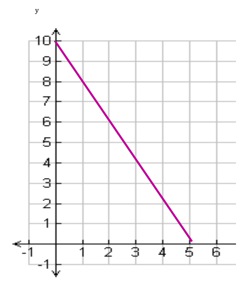
Step 2: Find b using (-6,5)

Ex 2: Find the rule of the line going through (-2,6) & (1, 3)

TOV Case 1- Both a and b are clear

Run | Rise

x	y
0	10
1	8
2	6
3	4



a= ___ b= ___ y =_________

TOV Case 2- a is clear, must find b

Run | Rise

x	y
5	37
7	47
9	57
11	67

a= __ b= ___ y =_________

Practice:
4.5 # 1-9

A table of values shows the relationship between two variables, typically x and y.

time	height
0	1.5
1	3
2	4.5
3	6

We call

X the independent variable because we choose it.

y is the dependant variable because it depends on the chosen value of x.

TOV Case 3- a not clear and must find b

Run | Rise

x	y
32	59
14	23
5	5
73	141

$\frac{\text{Rise}}{\text{Run}} =$

a= __ b= ___ y =________

Determine the **degree** of each function.

$f(x)= 2x+3$ ___1___ $f(x)= 5$ ___0___

$f(x)= 3x^2 -2x + 1$ ___2___ $f(x)= 2x^3 + 3x$ ___3___

$f(x)= -4x+1$ ___1___ $f(x)= \frac{1}{x}$ ___none___

The degree of a function determines the type of function

Degree	Type of function
0	Constant
1	Linear
2	Quadratic

Linear → Direct Linear function

Linear → Partial Linear function

Unit 4

L6 Types of Functions

- Linear Functions have degree 1

- The rule: $y = ax + b$

  or $f(x) = ax + b$

Case 1: if $b = 0$ → Direct Variation Linear function

Case 2: if $b \neq 0$ → Partial Variation Linear function

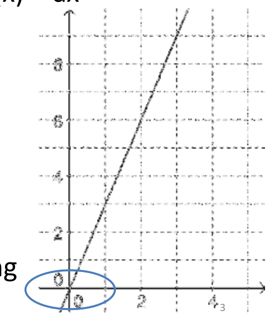
Case 1: if b=0 Direct Variation Linear function

Properties:

1. Every y-value is a direct multiple of the x-value
2. The rule: $y = ax$ or $f(x) = ax$
3. Table of values:

x	y
0	0
1	3
2	6
3	9

Constant ROC



4. Graph: a Diagonal line passing through origin (0,0)

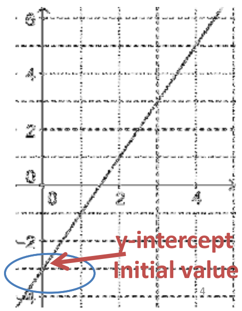
Case 2: if b≠0 Partial Variation Linear function

Properties:

1. y-values are not direct multiples of the x-values
2. The rule: $y = ax + b$ or $f(x) = ax + b$
3. Table of values:

x	y
0	-3
1	-1
2	1
3	3

Constant ROC



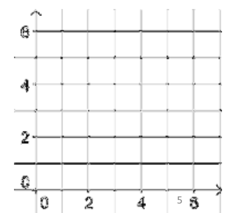
4. Graph: a Diagonal line passing through y-axis at point (0,b)

Constant Functions

Properties:

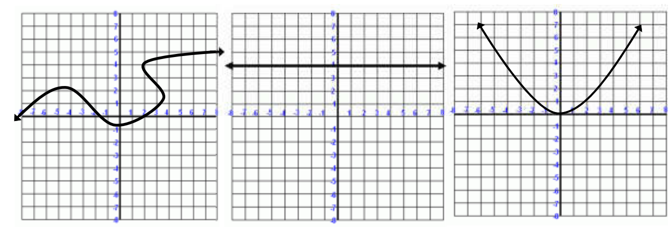
1. The ROC = 0 (∴aka: zero variation function)
2. The rule: $y = b$ or $f(x) = b$
3. Table of values:

x	y
0	5
1	5
2	5



4. Graph: Horizontal line passing through the y-axis at b.

Ex 1: is it Constant, Direct Linear, Partial Linear or Other



Ex 2: is it Constant, Direct Linear, Partial Linear or Other

x	y
1	3
2	5
3	7
4	9

x	y
10	20
9	18
8	16
7	14

x	y
0	5
20	25
40	45

Ex 3: is it Constant, Direct Linear, Partial Linear or Other

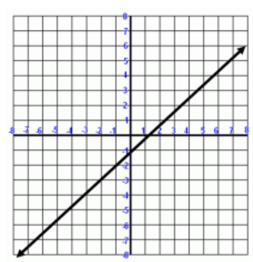
$y = 4x + 1$

$y = x^2$

$y = \sqrt{x}$

$f(x) = 3x$

$f(x) = 5$



Practice:
4.7 # 1-5

Unit 4

L7 Graphing and understanding Parameters a and b in y = ax + b

1) Make a Table of Values (min 3 points)

2) Choose easy x-values like ________.

If your slope (a) is a fraction, pick multiples of the value for the **run (denominator)**.

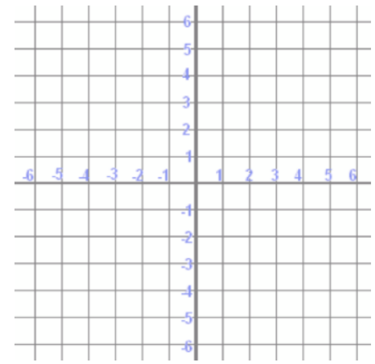
- ie if slope = $\frac{3}{4}$ pick x =0, 4, 8.

1. Graph and label these equations

$y = 3x$ $\quad y = -3x + 2$ $\quad y = \frac{1}{3}x$ $\quad y = -\frac{1}{3}x - 3$

$y = 3x$

$y = \frac{1}{3}x$



$y = -3x + 2$

$y = -\frac{1}{3}x - 3$

A faster way to graph a line straight from the rule:

The rule of the line is:

y = ax + b

b : y-intercept

a : rate of change = $\frac{rise}{run}$

2. Graph and label these equations

$y = 3x$ $\quad y = -3x + 2$ $\quad y = \frac{1}{3}x$ $\quad y = -\frac{1}{3}x - 3$

$y = 3x$

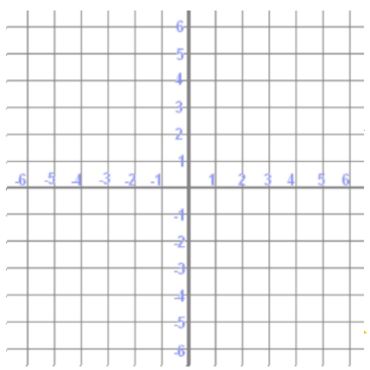
a = $\frac{3}{1}$ = $\frac{rise}{run}$

b = 0

$y = -3x + 2$

a = $\frac{-3}{1}$

b = 2



$y = \frac{1}{3}x$

a = $\frac{1}{3}$

b = 0

$y = -\frac{1}{3}x - 3$

a = $\frac{-1}{3}$

b = -3

Parameters a and b in y = ax + b

- For every line y = ax + b

(the parameters are a and b, and they each affect the look of the line)

- We can observe the affects of changing the parameters by using technology:

TI-83 or GeoGebra or graphsketch.com

A	f(x) = x	g(x) = 2x	h(x) = 0.5x
type of function			
R.O.C. (a)			
Initial value (b)			
Description of line			
B	**p(x) = -x**	**q(x) = -2x**	**r(x) = -0.5x**
type of function			
R.O.C. (a)			
Initial value (b)			
Description of line			

C	f(x) = x	s(x) = x + 2	t(x) = x – 4
type of function	**Direct**		
R.O.C. (a)	**1**		
Initial value (b)	**0**		
Description of line	**-Centered in 1st and 3rd quadrants. -Increasing line.**		
D	**u(x) = 3x + 2**	**v(x) = 0.5x – 4**	**w(x) = -2x + 6**
type of function			
R.O.C. (a)			
Initial value (b)			
Description of line			

Conclusions:

- For every line y = ax + b (the parameters are a and b affect the look of the line)
- a : affects the angle of inclination (steepness of line)
- b : affects the vertical translation of the line

Practice:
4.6 # 1-3

L8 Systems of 1st Degree Equations

- Finding the solution to a system of equations means find a common point (x,y), that fits into both equations at the same time.
- We can check the solution of a system by replacing it back into the original equations to see if it works.
- We can find the solution by making a table of values and finding when the values for y are the same.
- We can also graph the two lines on the same grid and see where the lines cross.

Ex 4: Solve using the comparison method

$y_1 = 4x + 6$

$y_2 = -2x$

Both equations must be isolated for the same variable

Compare both y's

Solve for x

Replace x and solve for y_1 then in y_2 to check

$\therefore$ *solution is*

Ex 5: Solve using the comparison method

$y = 3x – 2$

$y = 5x + 6$

$\therefore$ solution is (

Practice:

4.8 # 1-6

4.8 # 7-12

Unit 4

L9 Rational function

Savannah wants to repaint the offices at work. The job requires 40 hours of work for one employee. In this situation, consider the function f which associates the number x of employees hired for the job with the time y that it takes to complete the job.

a)

# emplo-yees x	Duration (in hours) y
1	
2	
4	
5	
8	
10	
20	
40	

b) as x increases the y decreases,

c) $\frac{10-20}{4-2} = -5; \quad \frac{8-10}{5-4} = -2$

so **R.O.C is not constant so it is not a line**

d) The **product** of x and y is **constant** (always 40).

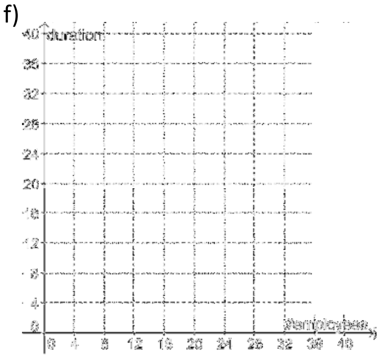
e) The rule is $y = \frac{40}{x} \quad or \quad f(x) = \frac{40}{x}$

L9 Rational function

Savannah wants to repaint the offices at work. The job requires 40 hours of work for one employee. In this situation, consider the function f which associates the number x of employees hired for the job with the time y that it takes to complete the job.

# emplo-yees x	Duration (in hours) y
1	40
2	20
4	10
5	8
8	5
10	4
20	2
40	1

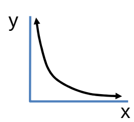
f)



L9 Rational Function

- Variables x and y are **inversely proportional**

  That is as x increases y decreases.
- The rate of change is **not constant**.
- The product of each pair x and y is **constant**.
- The rule is: $y = \frac{k}{x}$ because $xy = k$
- The graph looks like



x	y
2	6
4	12
5	15

x	y
2	14
4	8
5	5

x	y
2	50
4	25
5	20

Practice:
4.9 # 1-4